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Kalam Infinity Arguments and the Infinite Past

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1.0 Introduction

In their contribution to *The New Mormon Challenge: Responding to the Latest Defenses of a Fast-Growing Movement* (NMC) entitled "Craftsman or Creator: An Examination of the Mormon Doctrine of Creation and a Defense of *Creatio Ex Nihilo*," Paul Copan and William Lane Craig (C&C) raise two logical arguments in order to establish that the universe, in the sense of all that exists in any sense, must have begun to exist from absolute nothing a finite time ago.[1] They argue that an actual infinite series is impossible and therefore the universe cannot

1. Paul Copan and William Lane Craig, "Craftsman or Creator: An Examination of the Mormon Doctrine of Creation and a Defense of Creatio Ex Nihilo," in Francis Beckwith, Carl Mosser, and Paul Owen, eds., *The New Mormon Challenge: Responding to the Latest Defenses of a Fast-Growing Movement* (Grand Rapids, Mich.: Zondervan, 2001), 113–77. Though I focus on NMC, I will also discuss the extensive defense of the argument by Craig in *The Kalam Cosmological Argument* (New York: Harper & Row, 1979); with Quentin Smith, *Theism, Atheism, and Big Bang Cosmology* (Oxford, England: Clarendon Press, 1993); "God and Real Time," *Religious Studies* 26, no. 3 (1991): 335–47; "Time and Infinity," *International Philosophical Quarterly* 31, no. 4 (1991): 387–401; "Professor Mackie and the Kalam Cosmological Argument," *Religious Studies* 20, no. 3 (1985): 367–75; "The Kalam Cosmological Argument and the Hypothesis of a Quiescent Universe," *Faith and Philosophy* 8 (1991): 104–8; "A Swift and Simple Refutation of the *Kalam* Cosmological Argument?" *Religious Studies* 35, no. 1 (1999): 57–72; "Graham Oppy and the Kalam Cosmological Argument," LeadershipU, Novermber 8, 2005, http://www.leaderu.com/offices/billcraig/docs/oppy.html (accessed May 2, 2012); "Reply to Smith: On the Finitude of the Past," *International Philosophical Quarterly* 33, no. 2 (1993): 225–31.

be eternal. They also argue that it follows that the universe was created *ex nihilo* by a nonmaterial and yet personal being.

I argue, in response, that C&C's arguments do not apply to the order of infinity involved in an infinite past. Therefore, the two infinity arguments proposed by C&C are not sound. The first argument turns on an equivocation in the use of terms such as "number," "more than," and similar terms. In particular, the first argument mistakenly applies the meaning of terms used for finite mathematics such as "number," "equal to," and "more than" to transfinite set logic where these concepts mean something quite different. The first argument commits the fallacy of equivocation by adopting the logic that applies to individual members in finite collections to infinite sets. I also argue that neither premise of the second argument applies to the order of infinity involved in the infinite past and is, therefore, based on two false premises.

I further argue that the two infinity arguments do not establish a logical contradiction in the concept of an actually infinite series that has no first term—as C&C readily admit. However, I show that their arguments can be sound only if the coherence of Cantor's theories of infinite set logic is called into question. I then argue that it can be demonstrated that it is *logically possible* that a material universe has always existed without beginning. I argue that the alleged "absurdities" claimed by C&C are perhaps unusual—given the fact that our experience is limited—but that they are neither impossible nor absurd.

Finally, I argue that, even if they were sound, the infinity arguments do not apply to temporally discontinuous temporal epochs, each of which is finite but infinite as a collection. Thus, the arguments do not apply to the discontinuous temporal epochs posited by the quantum vacuum and chaotic inflationary theories of the universe.

2.0 The Nature of Infinities

C&C do not argue that the notion of infinity is incoherent. They admit that Cantor's mathematical logic of infinite sets is self-consistent. Indeed, they acknowledge that "the actual infinite may be a fruitful and consistent concept within a postulated universe of discourse." However, they claim that, "it cannot be transposed into the real world, for this would involve counter-intuitive absurdities."[2]

2. NMC, 150.

Mathematicians have been comfortable reasoning about the infinite for some time. However, prior to the breakthrough work of Georg Cantor, mathematicians refused to consider infinities in their theories.[3] The reason for this reluctance was simple and straightforward: for any inductive[4] finite number n, the number n + 1 has two certain properties: First, $n^{1} n + 1$. Second, $n < n + 1$ and $n > n - 1$. However, they refused to accept mathematical infinities because infinite collections violate these simple rules, which seemed to be both contrary to logic and absurd. As Bertrand Russell notes,

> The difficulties that so long delayed the theory of infinite numbers were largely due to the fact that some, at least, of the inductive properties were wrongly judged to be such as *must* belong to all numbers; indeed it was denied without contradiction. The first step in understanding infinite numbers consists in realizing the mistakenness of this view.[5]

The most amazing difference between an inductive number and an infinite number is that the rules that apply to finite "numbers" do not apply to infinite "numbers." The word "number" is thus equivocal when used for infinite sets rather than finite sets. Consider the set of all cardinal numbers: 0, 1, 2, 3, 4, 5 . . . This set has a first member but no last member. It is infinite. Cantor called the smallest of infinite cardinals $\aleph_0$ (aleph-zero). This "transfinite number" has some very different properties from finite inductive numbers. We can add or subtract 100,000 to or from $\aleph_0$ and it is the same transfinite number! That is, it still has the property of being an infinite set. Indeed, we can add any finite number to $\aleph_0$ and it is still the same transfinite number. Indeed, we can add $\aleph_0 + \aleph_0 = \aleph_0$. Cantor adopted the fact that 1 can be added to $\aleph_0$ as a definition of transfinite numbers, so $\aleph_0 + 1 = \aleph_0$. However, as I will discuss below, it is better to include within the definition of transfinite cardinal numbers the recognition that they are "numbers" that do not possess all proper-

3. Georg Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers*, trans. O. Jourdain (Chicago: Open Court, 1915).

4. By "inductive number" I mean any number that has the inductive properties of mathematics roughly in the sense argued by Bertrand Russell and Alfred North Whitehead in *Principia Mathematica* (Cambridge: Cambridge University Press, 1910–13).

5. Bertrand Russell, *Introduction to Mathematical Philosophy* (1919; rpt., New York: Dover Publications, 1993), 78–79.

ties of inductive numbers (i.e., transfinite numbers are not inductive numbers).

This fact is important because C&C attempt to exploit our intuitions about finite numbers and argue that it is absurd that infinite numbers do not act like finite numbers. Indeed, they refuse to accept the possibility of an actual infinite for the same reason that mathematicians so long refused to accept transfinite numbers: they do not obey the rules that apply to finite sets. But this difference between properties of finite numbers and transfinite numbers arises because transfinite numbers actually define properties of sets and not of individual members of sets, as do inductive numbers. Sets often have different properties than their individual members. Thus, we might say that a large crowd of people is not the same as a crowd of large people.

However, not all transfinite numbers have the same "number" of terms or the same properties. That is, not all transfinite number have $\aleph_0$ terms. The number of the set of real numbers is "greater than" $\aleph_0$; in fact, it is $2\aleph_0$. This result follows by considering all of the subclasses of $\aleph_0$. If a class has n members, it contains 2^n subclasses—that is, there are 2^n ways of configuring its subclasses. Further, there is no maximum to the infinite cardinal numbers. However great the number in any infinite set n, the number 2^n will always be greater.

Now the arithmetic of transfinite numbers is a bit surprising until it is grasped that it is the property of infinity that generates these interesting results:

$\aleph_0 + 1 = \aleph_0$
$\aleph_0 + n = \aleph_0$ for any n that is a finite inductive number
$\aleph_0^2 = \aleph_0$

However, it must be noted that

$2^{\aleph_0} > \aleph_0$

Further, although addition and multiplication work well for transfinite numbers, we cannot obtain definite results for subtraction and division. Because transfinite numbers have different properties than finite numbers, we obtain different results. Subtraction of a finite number from an infinite number is straightforward:

$\aleph_0 - n = \aleph_0$ for all n that are inductive numbers

However, $\aleph_0 - \aleph_0 = 0 \ldots \aleph_0$

That is, when we subtract $\aleph_0$ from itself, the result is not definite. Consider the results of subtracting the collection of $\aleph_0$ from the following:

a) All inductive numbers – $\aleph_0 = 0$
b) All inductive numbers from n onwards – $\aleph_0$ = remainder in numbers 0 to n – 1
c) All odd numbers – $\aleph_0$ = all even numbers

All of these ways of subtracting $\aleph_0$ from $\aleph_0$ give different results. Division is similar. Whenever $\aleph_0$ is multiplied by any finite number, the product is always $\aleph_0$ terms. However, $\aleph_0$ divided by $\aleph_0$ may have values ranging from 1 to $\aleph_0$. It follows that negative numbers and ratios do not apply to transfinite numbers. C&C complain that to "avoid the contradictions involved in subtraction of infinite quantities, transfinite arithmetic simply prohibits such inverse operations by fiat."[6] But this assertion is simply erroneous, for transfinite mathematics does not simply prohibit subtraction and division by fiat; rather, transfinite mathematics shows why operations that can be done with finite numbers sometimes give indefinite results for transfinite numbers. It should be noted that Cantor's theory is not the only systematic exposition of transfinite logic. Graham Oppy has pointed out that there are a number of other developed theories of transfinite numbers that deal with inverse operations with transfinite numbers without contradiction.[7]

There are also different orders of transfinite numbers. In fact, the ordinal series of the form 1, 2, 3, 4 . . . n . . . represents the smallest of transfinite serial or ordinal numbers, which Cantor called w. Moreover, various serial successions may be greater than others. The ordinal number of the series of all ordinals that can be made out of a $\aleph_0$ collection, taken in order of magnitude, is called ω_1. Moreover, it can be shown that $1 + w\ ^{1}\ w + 1$. Such a rule is true of all relation-numbers and not merely transfinite numbers. If m and v are two relation-numbers, the general rule is that $m + v\ ^{1}\ v + m$. Thus, when discussing transfinite orders, it is essential to note that differing orders have different values. The infinite order collection beginning with {1 + 0, 1, 2, 3 . . .} has a different value than the set (. . . –3, –2, –1, 0 + 1). Thus, the order of the

6. NMC, 155.

7. Graham Oppy, "Reply to Craig: Inverse Operations with Transfinite Numbers and the Kalam Cosmological Argument," *International Philosophical Quarterly* 35, no. 2 (1995): 219–21.

past without a beginning to which it is added has a different order value than the infinite past that is counted down to zero.

The series of, first, all odd numbers, and then all even numbers, has a serial number of 2ω. This number is "greater than" $\aleph_0 + n$, where n is any finite number.

There is also a crucial distinction between a "well-ordered" series and a "not-well-ordered" series. A well-ordered series has a *beginning*, and has *consecutive terms*, or has a *next term* after any selection of terms, like {1/2, 1/4, 1/8, 1/16, 1/32 . . .}. A "not-well-ordered" series is one that *has no first term or is discontinuous*. The series of negative numbers beginning with –1 and counting backward {–1, –2, –3 . . .} is well-ordered. However, the series *ending in* –1 in a series {. . . –4, –3, –2, –1} is not-well-ordered. The reason this is important is that not-well-ordered series do not have the same properties as well-ordered infinite series. The infinite past has the cardinal number $\aleph_0$ and the order type ω_0. Not-well-ordered series do not obey the commutative law [(a + b = b + a) and (a + b = b x a)] or the distributive law [a (b + c) = ab + ac]. The distributive law holds for transfinite ordinals in the form

$$(b + c)\, a = ba + ca$$

but does not hold for the form

$$a\, (b + c) = ab + ac$$

Now we must ask, what does it mean for two sets to have the same number of members? For finite numbers, the two sets simply have the same finite number, say 4. That tells us how many are in each set. However, for set theory, two sets have the same "number" of members if they can be put into a one-to-one correspondence with one another. Thus, these two sets have the same number of members:

a	b	c	d
w	x	y	z

An infinite set is one whose proper subset can be put into a one-to-one correspondence with the whole of the set. Consider the set of count numbers and odd numbers:

Count numbers: 1 2 3 4 5 6 7 8 . . .
Odd numbers: 1 3 5 7 9 11 13 15 . . .

Note that no count numbers are left over, so no odd number is paired with more than one count number. There is a one-to-one correspondence. Thus, the sets of count numbers and of odd numbers are both infinite sets. However, take any finite collection of numbers:

{2, 4, 6, 8}

No proper subset of this set can be put into a one-to-one correspondence with the whole. Thus, the set is finite. In other words, the set {2, 6, 8} cannot be put into a one-to-one correspondence with the set: there will always be something remaining. Thus, we can adopt the following rules regarding finite and transfinite numbers:

> *R1* = A finite set has more members than any of its proper subsets
>
> *R2* = An infinite set does not have more members than any of its proper subsets, and each member of an infinite set can therefore be placed into a one-to-one correspondence

For finite sets, the whole set is always "greater than" a subset consisting of only some but not all of the set's members. For an infinite set, the whole set is not "more than" a proper infinite subset consisting of only some but not all of the set's members.

It is imperative to see that transfinite sets have different properties than finite sets. Thus, when we use terms like "number," "greater than," and "equal to," they mean something different for a transfinite series than for a finite series. In transfinite logic, for two transfinite sets to have the same number of members means that the members of each infinite collection can be placed in a one-to-one correspondence. However, for two inductive numbers to be equal means that they are "the same number."

3.1 The First Infinity Argument

C&C begin by distinguishing between a potential infinite and an actual infinite. A potential infinite is one that is always actually finite but open-ended without limit. All of the members of the collection do not yet actually exist (and thus technically do not form a set). An actual infinite series is one that has an actually infinite number of members. C&C maintain that if the universe has always existed, then, for example, the series of events up to the birth of Cantor constitutes an *actual* infinite. However, the series of events since Cantor's birth and stretching into the future, *provided that* the future is unending, constitutes only a *potential* infinite. C&C argue that a potential infinite can exist in the real world, but an actual infinite cannot. The distinction is one of the ontological statuses of events.[8] Given these distinctions, C&C's first argument runs as follows:

> 1.1 An actual infinite cannot exist.
> 1.2 An infinite temporal regress is an actual infinite.
> 1.3 Therefore, an infinite temporal regress of events cannot exist.

The first premise is obviously the important premise in the argument—although I want to begin by arguing that premise 1.2 is ambiguous and problematic. We may ask what it means for a series of past events to "be actual," for it seems that the past is not still actual, and an infinite series can be *actually infinite* only if all of its temporal moments are actual at once. Those who adopt an A-theory of time maintain that there is a genuine distinction between past, present, and future and that the past and future are not actual but only the present moment is actual. On the other hand, those who adopt a B-theory of time maintain that the past, present, and future are equally real or actual. C&C seem to want to make an ontological distinction between a potentially infinite series and an actually infinite series. The difference is that the events in an actually infinite series are ontologically real; they actually exist in the real world are not merely mental or mathematical constructs—as C&C take numbers and mathematics to be. However, an infinite temporal regress is not the same as a beginningless series of events in time. An infinite temporal *regress* constitutes a "well-formed" infinite series. It has a beginning term, 0 (or −1, depending on how the set is constructed), and then counts backwards {0, −1, −2, −3 . . .}. An infinite temporal regress has the same mathematical properties as the set of real numbers beginning with 0 (or 1) and counting forward. However, because a be-

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8. The distinction between an actual and a potential infinite is made by virtually every proponent of kalam infinity arguments. See G. J. Whitrow, *The Natural Philosophy of Time*, 2nd ed. (Oxford, England: Clarendon Press, 1980), 200; "Time and the Universe," in *The Voices of Time*, ed. J. T. Fraser (New York: George Braziller, 1966); and "On the Impossibility of an Infinite Past," *British Journal for the Philosophy of Science* 29, no. 1 (1980): 31–32; Pamela Huby, "Kant or Cantor: That the Universe, If Real, Must Be Finite in Both Space and Time," *Philosophy* 46, no. 176 (1971): 127; David A. Conway, "Possibility and Infinite Time: A Logical Paradox in St. Thomas's Third Way," *International Philosophical Quarterly* 14, no. 2 (1974): 201–8.

ginningless series of events is a "not-well-formed series," it has different mathematical properties. Recall that the series of past events *ending* with {. . . –4, –3, –2, –1, 0} has different mathematical properties than the set *beginning* with {0, –1, –2, –3, –4 . . .}.

Perhaps C&C would maintain that since the past has been real—it has actually existed in reality—it constitutes an *actual infinity* of events even as regress. However, this assertion is ambiguous. An "actual infinity" can mean that (a) an infinite set of events are all actual at once; or (b) an infinite set of events, some of which have been, are no longer actual. The infinite past does not constitute an infinity of *actual events.* In particular, the set of past events does not constitute a set of *actual events* because the past events are no longer actual (assuming an A-theory of time). This distinction becomes important when setting up stories that supposedly show that a beginningless reality is absurd. After all, if the universe (in the sense that it constitutes all that is) has always existed, then it constitutes a beginningless series but in actuality is not a regress of events.

Nevertheless, premise 1.2 is easily repaired by replacing it with one that accurately mirrors the conditions of a beginningless reality:

> 1.2* A beginningless series of events in time is an actual infinite.

From 1.1 and 1.2*, it follows that:

> 1.3* A beginningless past series of events in time cannot exist.

With this correction, we can assess the first argument. C&C attempt to show that premise 1.1 is true by a *reductio ad absurdum*. They suppose for the sake of argument that an actual infinite series exists and then proceed to attempt to derive absurdities from that assumption. In NMC, they use the example of Hilbert's Hotel, derived from the work of the mathematician David Hilbert. Suppose we have a Hotel with an infinite number of rooms and that "all" of the rooms are "full." Now suppose that a new guest arrives and asks for a room. Is there room? Of course, because remember that $\aleph_0 + 1 = \aleph_0$. C&C protest: "But remember, before he arrived, all of the rooms were full! Equally curious, according to the mathematicians, there are now no more persons in the hotel than there were before: the *number is just infinite*. But how can this be? The proprietor just added the new guest's name to the register and gave him the keys—how can there not be one more person in the hotel than before?"[9] C&C then suggest that we subtract one guest, or move all of the occupants to only even-numbered rooms and thus open up an infinite "number" of rooms for an infinite number of new guests in odd-numbered rooms, or have the odd-numbered guests move out and move the even-numbered guests back again—and we always end up with the same "infinite" number of guests. C&C quip: "Can anyone believe that such a hotel could exist in reality?" C&C conclude that the existence of an actual infinite is absurd and therefore impossible. However, these absurdities are all in their own minds.

We may ask if the example of Hilbert's Hotel is really an analogy for the type of infinity involved in a beginningless past? The answer is that it is not. It does not mirror the infinite past. First, Hilbert's Hotel has a first room, a beginning term, that is followed by consecutive terms and therefore is a well-formed series. The infinite past is a *not-well-formed* infinite series because it has no first term. Second, the rooms in the Hotel all exist at once and are actual in the same moment. That is not true of the infinite past. Only the present moment is actual or ontologically real, assuming an A-theory of time (which C&C accept). Thus, the past events do not actually exist to be transposed and reordered, as the story of Hilbert's Hotel requires. If Hilbert's Hotel were like the past, it would have only one room that has been occupied by an infinite number of guests in consecutive order. Further, the past cannot be jumbled around like the persons in Hilbert's Hotel for reasons quite unrelated to the problems of infinities—*the past is fixed and unchangeable once it occurs*. Year 351,067 B.C. cannot be exchanged for year 465,789 B.C. Thus, we cannot take away all of the odd years. We have the infinite series of past events just as they have occurred and in the very order they occurred, and we cannot alter them in the way C&C suggest for Hilbert's Hotel to create a supposed absurdity.

For these reasons, the supposed "absurd" stories used by C&C to demonstrate that an actual infinite is absurd simply have no application to the type of infinite order involved in the past without a beginning. All of the supposedly absurd stories—like Hilbert's Hotel, or the Tristram Shandy autobiography[10]—depend critically upon properties of the

9. NMC, 151.

10. See the discussions of the Tristram Shandy paradox in Craig and Smith, *Theism, Atheism, and Big Bang Cosmology*, 33–35. A. R. Small notes that the Tristram Shandy story only gives us half of the supposed paradox. See A. R.

order that the infinite past does not possess. The past events are not like an infinite number of guests in an infinite number of existing rooms, all of which actually exist in the same moment that can be shuffled around and transposed and still maintain the same order of infinite numbers.

Further, the supposed absurdity is contrived. Take the first supposed absurdity—that the number of occupied rooms equals the number of rooms plus one for a new occupant, and that there are no "more" occupied rooms than there were before the new occupant arrived. Absurd? Not really. C&C illicitly use the concepts of "number" and "more" to trade on our intuitions about finite numbers and then apply them to transfinite numbers where such intuitions do not apply. It *is* absurd to suggest for a finite number that 99 rooms plus 1 more room equals the same number of rooms as before. However, in the context of infinite set logic, all infinite collections can be put into a one-to-one correspondence with proper subsets of themselves, and so our ordinary expectations about the way finite numbers behave do not apply in this new context. To say that the "number" of rooms is "infinite," even after a new guest has checked in, is to say only this and nothing more: the occupied rooms before the new guest arrived can be put into a one-to-one correspondence with all counting numbers, and so can the number of occupied rooms after a new guest arrives and one more room is occupied.

Is it absurd to suggest that we can have a Hotel that is full and then move all of the occupants to even-numbered rooms and leave an infinite number of odd-numbered rooms for an infinite number of new guests? Hardly. Conceive of the Hotel as extending into infinity from a certain point in Denver, which is the beginning of the rooms. There is a first *even*-numbered room, a second even-numbered room, a third even-numbered room, and so forth without limit. But there is also a first *odd*-numbered room, a second odd-numbered room, and so forth without limit. There is also an even-numbered room that can be placed into a one-to-one correspondence with every counting number—and the same for all odd-numbered rooms. But that is just what it means to say the number of even-numbered rooms equals the num-

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Small, "Tristram Shandy's Last Page," *Bristish Journal for the Philosophy of Science* 37 (1986): 213–16. The response to Craig in Craig and Smith in found in *Theism, Atheism, and Big Bang Cosmology*, 88–89. Wes Morriston also responds to Craig's Tristram Shandy argument in "Must the Past Have a Beginning?" *Philo* 2, no. 1 (1999): 5–9.

ber of odd- and even-numbered rooms together. Perhaps a better word than "number" could be used, for we can say instead that the number of even-numbered rooms alone *corresponds* to the number of even- and odd-numbered rooms taken together. Once we clear up the equivocal use of the word "number," the supposed absurdity evaporates. C&C suggest that it is absurd that the "number" of occupied even- and odd-numbered rooms can correspond to just the odd-numbered rooms, but that is the way that transfinite numbers work. In fact, once we state that even-numbered rooms can be put into one-to-one correspondence with even- and odd-numbered rooms taken together, the assertion becomes quite ordinary and mundane. Indeed, one could not reject such a statement without simply objecting to Cantor's theory of transfinite mathematics altogether.

Yet there is a remaining feature that may make us feel uncomfortable. If there are "more" rooms than there are just even-numbered rooms, then the even-numbered rooms are a proper subset of the set of all rooms. Not every room in the Hotel is an even-numbered room. However, it is simply an error to assert that there are "more" rooms than even-numbered rooms because transfinite numbers follow different rules than finite numbers. Unless Cantor is simply mistaken, the number of rooms is not "more than" the number of even-numbered rooms for infinite sets. C&C claim that they have no objection to the abstract mathematical notions of an infinite series of numbers; they only object that such things cannot exist in reality. But the objection made by C&C is valid only if the theory of abstract transfinite numbers is wrong as well. When we subtract all odd numbers from all counting numbers, we have a set that has the same number of terms. Why is this not absurd when applied to abstract numbers of rooms but somehow becomes absurd and unthinkable when applied to real rooms? C&C's objections are really objections to the very notion of infinite numbers and not merely to whether such numbers can be mirrored in reality.

In effect, C&C are suggesting that an actual infinite is impossible because we can derive a violation of the following principle:

R_1 = A set has more members than any of its proper subsets.

They argue that a Hotel that has an infinite number of rooms that are all full and then has room to add an infinite number of guests is impossible. The reason it seems impossible is that, for finite numbers, a set has "more members" than any of its proper subsets. But what does it mean to claim that actually infinite sets have a "number" of members? An actually infinite set does not have an inductive number, or a natural number, or a real number of members. Thus, it is simply mistaken to assert that the Hotel has a "greater inductive number," or a "greater natural number," or a "greater real number" of occupants. Actual infinities have a *transfinite number* of members, and transfinite numbers do not obey R_1, for R_1 is a rule that applies only to finite numbers. Rather, transfinite numbers obey:

> R_2 = An infinite set does not have more members than any of its proper subsets.

Thus, it is not true that actually infinite sets have a greater *transfinite* number than all of their proper subsets.

Now, C&C may claim that it is simply impossible for "a set Z to have every member that another set Y has, and also has some 'more' members that the set Y doesn't have, and yet set Z does not have 'more members' than set Y." Yet this assertion is true for transfinite numbers and not for finite numbers. There is a sense in which set Z has "more" members than set Y, but it is a sense that applies only to finite numbers and does not affect the fact that both Z and Y are infinite sets. When we say that an infinite set has every member that another set has and some "more" members in addition, "more" means only that "set Z has all of the members of Y and some members that Y doesn't have and both are infinite sets." The fact that set Z has all of the members of set Y and also some members that Y doesn't have does not preclude the members of set Y and of set Z from being placed into a one-to-one correspondence if Y and Z each has an infinite number of members. When infinite sets are compared, the word "more" does not mean the same thing that it does when finite sets are compared.[11]

Thus, this first argument commits the fallacy of equivocation in the sense that it imputes the properties of individual members of a

11. Wes Morriston, John Taylor, and Paul Draper have also noted that Craig surreptitiously imputes the properties applying to finite numbers to transfinite numbers. See Wes Morriston, "Must the Past Have a Beginning?" *Philo* 2, no.1 (1999): 5–19; and "Craig on the Actual Infinite," *Religious Studies* 38, no. 2 (2002): 147–66; John Taylor, "Kalam: A Swift Argument from Origins to First Cause?" *Religious Studies* 33, no. 2 (1997): 167–79; and Paul Draper, "A Critique of the Kalam Cosmological Argument," in *Philosophy of Religion*, ed. Louis Pojman, 3rd ed. (Belmont Calif.: Wadsworth, 1997), 42–47.

finite set to infinite sets as a whole. Finite sets obey Rule R_1. However, it is a mistake to impute this same rule to the properties of infinite sets. Sets do not necessarily obey the same rules that apply to individual members of sets.

3.2 The Second Infinity Argument

C&C offer a second infinity argument that is even weaker than the first:

> 2.1 The temporal series of events is a collection formed by successive addition.
> 2.2 A collection formed by successive addition cannot be an actual infinite.
> 2.3 Therefore, the temporal series of events cannot be an actual infinite.

Premise 2.1 is not true of all temporal series and certainly is not true of a beginningless past series that terminates in the present. An actual past infinite collection is not "formed by successive addition," as if we could add a finite number of terms together and somehow they add up to an infinite number. Rather, for any term added to the past at any given point in the past, the past is *already* an infinite collection at that past time and therefore is not *formed as an infinite* collection by such addition. Indeed, C&C have assumed in premise 2.1 that the "temporal series" of past events has the same properties as a potential infinity rather than an actual infinity, for they assume that an infinity is open-ended and completed by adding to it rather than being a completed infinity without a beginning term. One cannot form a collection by adding to it if the collection already exists *before* the addition. The infinite past already exists as an infinite temporal series before the addition of any term and therefore cannot be formed as an infinite temporal series by addition. I accept that one cannot *by beginning with any one member* of an infinite set complete an infinite set by successively adding new members to the set. But what follows from this is only that a set *that has a first member*, that is, one that could be a "well-formed set," cannot be completed as an actual infinite by successive addition. However, premise 2.1 is not true for "not-well-formed" sets that have no first member. The view that the past consists of an infinite series without beginning does not imply that it can become or "be formed as" an infinite collection by successive addition.

Indeed, C&C quote Bertrand Russell: "Classes which are infinite are given all at once by the defining properties of their members, so there is no question of 'completion' or of 'successive synthesis.'"[12] They take this to support premise 2.2. However, they miss the entire point of Russell's statement. Russell is saying that it is a mistake to assert that infinities of any sort are created or completed by successive addition, which is precisely what premise 2.1 implies. Premise 2.1 is false because it mischaracterizes the properties of an actual past infinite. The past is not "formed" as infinite by adding new days or years to it—it is already infinite at any given moment a new day or year is added! While there is never enough time to add up finite numbers to an infinite in a finite amount of time, the number of times is always already infinite if there is no beginning term—and thus the events constitute members of an actually infinite series. However, there is no time at which the temporal series of events *became* or were *formed* as an infinite collection. There is no sense in which an infinite is "formed" or "completed" by addition, just as Russell says. We cannot form an infinite set by adding 1 or 1,000,000 to it unless the set to which the new number is added is already infinite. Thus, premise 2.1 is false and the argument fails.

However, C&C may claim that the fact that an infinite series cannot be formed by successive addition merely shows that it cannot be a temporal series that can be added to by temporal succession. This is because temporality is essentially defined by the fact that one event is preceded and followed by another. Why should we accept that an infinite temporal series must be formed by successive addition? Craig claims that it is the very nature of all temporal series that they must be formed by successive addition:

> The only way a collection to which members are being successively added could be actually infinite would be for it to have an infinite "core" to which additions are made. But then it would not be a collection *formed* by successive addition, for there would always exist a surd infinite, itself not formed successively but simply given, to which a finite number of successive additions have been made. But clearly the temporal series of events cannot be so characterized, *for it is by its very nature successively formed throughout*. Thus, prior to any arbitrarily

12. NMC, 156–57.

> designated point in the temporal series, one has a collection of past events up to that point which I successively formed and completed and cannot, therefore, be infinite.[13]

This assertion is no response at all. First, note that the proponent of the infinite past accepts the first three statements made by Craig. As Craig admits, an infinite collection can be added to by successive addition if it has a core that has always existed as an eternal past. Second, the collection is not formed as an infinite by successive addition but exists as an infinite past without some prior explanation for some first event. Further, a temporal series is such that it has events, and for each event there are events prior to and also events subsequent to the event. Even so, Craig argues that this cannot be because it is in the nature of an infinite past that it must be *formed as a series of events* by successive addition if it is a temporal collection. But he merely asserts this without further proof or argument. In fact, as Bertrand Russell states, this last assertion is false. It is in the very nature of the infinite past that it is always already infinite at any point it is added to. Thus, it is the nature of the infinite past that it is not formed as an infinite temporal collection by successive addition. Moreover, it won't do to observe that this view commits a person to the view that the eternal past has just always existed as an infinite, for that is just what the proponent of the eternal past claims. Craig cannot claim that such a view is absurd because an infinite cannot be added to, for he admits that there is one type of infinite that can be added to, and it just happens to be the very type of infinite claimed by the proponent of an eternal past. Finally, the claim that the entire temporal series itself must be formed as an infinite series by successive addition if individual members can be added to the series commits the fallacy of composition. It does not follow from the fact that individual members of a series have the property of being formed as part of an infinite series by being added to the series that the series set itself has the property of being formed as an infinite collection by successive addition.

A temporal series, by nature, can indeed be added to one member at a time, but it does not follow that an infinite temporal series is *formed as an infinite series* by such addition. Thus, premise 2.2 is ambiguous with respect to infinite series. It could mean that

13. Craig and Smith, *Theism, Atheism, and Big Bang Cosmology*, 34.

> 2.1* An infinite temporal series of events is formed *as an infinite temporal series* by successive addition.

On the other hand, it may be construed that

> 2.1** An infinite temporal series of events has been formed and is such that it can be added to by successive addition.

Premise 2.1* is false, while premise 2.1** may be true depending on the meaning of "has been formed." Now it is clear that if "has been formed" means anything like (a) "has been created at some first time," then it must be false because no one, not even God, can create a beginningless infinite temporal series of events having a first event. However, if "has been formed" means (b) "exists" or "has been created by God by forming events in each temporal moment of a past without beginning," then premise 2.1** is possibly true. It is indeed possible to add to a temporal series that has a transfinite number of members, but it is not possible to "form the series" *as an infinite series* by addition. However, if premise 2.1** in sense (b) is true, then premise 2.2 is necessarily false with respect to the transfinite order type w + *n*, where *n* is any finite number. That is, if the infinite temporal series of events is in fact formed as an actual infinite *in the sense that God has created it by forming events in each successive temporal moment of an eternal past*, then there is a series that is an actual infinite that is constituted as a temporal series by successive addition. This series is temporal in the sense that each moment is preceded by a prior moment and followed by a successive moment, but it is not infinite by virtue of the fact that each moment is succeeded by another moment; rather, it is infinite in virtue of the fact that there is no beginning to the series of succession.

Premise 2.2 also confuses the properties of a potential infinite with an actual infinite. C&C argue:

> In order for us to have "arrived" at today, existence has, so to speak, traversed an infinite number of prior events. But before the present event could arrive, the event immediately prior to it would have to arrive; and before that event could arrive, the event immediately prior to it would have to arrive; and so on *ad infinitum*. No event could ever arrive, since before it could elapse there will always be one more event that had to have happened first. Thus, if the series of past events were beginningless, the present event could not have arrived, which is absurd![14]

14. NMC, 156.

Indeed, the scenario painted by C&C is absurd because it does not accurately characterize the nature of the infinite past and its relation to the present. Just what does it mean to "traverse" an infinite time or to "arrive" at the present? If *traverse* means to pass through or complete a temporal series beginning with an event and ending with an event—as I believe the term implies—then the infinite past cannot be traversed in this sense.[15] However, the argument then would not apply to the infinite past since the infinite past has no beginning term. In fact, this seems to be the meaning of "traverse" implied in C&C's argument. Note first that C&C treat the past once again as a well-ordered infinity that has a first member—the first member of this set is the present event and we begin this set by counting backwards into an infinite past. We begin the thought experiment proposed by C&C by thinking of the present event, and then regress to the event before that, and then the event before that, *ad infinitum*. Since we begin with the present event, the infinity is merely potential and is in fact never completed. In fact, it is no infinity at all but merely an open-ended finite series. Since no matter how long we count we cannot complete the infinite past, C&C conclude that the past cannot be infinite. But counting backward again treats the actual past without beginning as a set that has a beginning term (i.e., the present event).

There is a way, and perhaps only one way, to create an actual infinite by counting or marking each successive moment, and that is to have been counting in each moment of existence of the eternal past as it occurred. Thus, the argument is a *non sequitur*, for it does not follow from the fact that the past is infinite that it cannot "reach" the present because there has been an infinite time in which to do it! Nor does it follow that no event could ever arrive because there will always be one more event that theoretically had to happen first. All that follows is that there is in fact an event that preceded the present event, and an event before that, and so on.

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15. Nicholas Everett faults the Kalam argument based upon *traversing, counting, moving across,* and *completing a task* because it presupposes an "empirical interpretation of 'pairing' and 'correlating' [that] is out of place when we are considering infinite sets." See Nicholas Everett, "Interpretations of God's Eternity," *Religious Studies* 34, no. 1 (1998): 29. Richard Sorabji also criticizes the "traversal argument" on several grounds. Richard Sorabji, *Time, Creation and the Continuum* (Ithaca, N.Y.: Cornell University Press, 1983), 219–24.

In another article, Craig protests that his traversal argument does not implicitly presuppose a beginning term. He says,

> It is not obvious to me that to say a beginningless infinite series cannot be traversed means that it has no first member. The best I can make [of this] claim is that the notion of traversal entails a beginning point, so that a series with no beginning point cannot be traversed. But such a construal of traversal seems clearly wrong. A man who just finished counting all of the negative numbers, for example, has 'traversed' a beginningless, infinite series. To traverse a series means just to cross it or pass through it one member at a time. Hence, I am quite at a loss to understand how the Kalam Cosmological Argument begs the question by assuming implicitly that the past has a beginning point.[16]

But this protest of surprise isn't really a response at all. C&C implicitly assume that the series of past events has a starting point by asking us to imagine the past as a series that begins with the present event as the first term of a regress and counting backwards. They give us a well-formed infinite series when they must give us an example of a not-well-formed series that has no beginning term and terminates in the present. Moreover, the very activity of counting assumes that a beginning number has started a count. However, let's take C&C at Craig's word that to traverse means to "cross" or "pass through" a series one member at a time. If that is all that traverse means, then there is no reason why an infinite series cannot be formed by successive addition in this sense, for all it means is that an infinite time has been "crossed" or "passed through" in an infinite number of days or years, each of which has been passed through one member at a time. There is nothing absurd or impossible about that.

C&C suggest that the infinite past is like the task of completing a countdown of negative numbers and ending at 0, which seems impossible. And why is it impossible? If one has existed in every moment of the infinite past, he could have counted a number for each past moment because there is a one-to-one correspondence between the past times and the past numbers counted. That, of course, is not impossible. Yet C&C counter,

> But that only pushes the problem back a notch: how could an infinite series of moments elapse sequentially? If the past is actually infinite,

16. Craig, "Graham Oppy and the Kalam Cosmological Argument."

> then why did one not finish his countdown of the negative numbers yesterday or the day before, since by then an infinite series of moments had elapsed? No matter how far along the series of past moments one regresses, one would have already completed his countdown.[17]

But there really is no problem at all. Our (finite) experience of counting includes the necessity that we begin counting with some number, but that is not the case if we set up the thought experiment correctly. There is confusion in the example suggested by C&C that is important to note. First, they suggest that the countdown ends with zero. Yet they say that the counter is regressing as he counts down, which suggests that the counter has begun at zero and is counting down the negative numbers backwards. But if we follow the example suggested by C&C in a way that actually reflects the infinite past, then there is no beginning to the counting. If there is no beginning to the counting, and in each new moment the counter counts a new number, it follows that the counter could have counted all of the negative numbers ending with zero, for there would then be a one-to-one correspondence between the infinite number of times in the past and each act of counting. Say that in fact I had been counting for all eternity. What follows is that I would have finished counting an infinite number of terms yesterday, and when I counted one more today I would have finished counting an infinite number of terms again today. As Wes Morriston argues in response to a similar argument by Craig:

> It is true that yesterday the infinite counter would have counted *infinitely many* numbers. Indeed, it is true that on any day during his count he would have counted infinitely many numbers. But it does not follow that on any day prior to today he has *finished* his count. Why? Because he was counting down to *zero*, and on no day prior to today had he reached zero. Yesterday, he had only reached –1, the day before he had reached –2, and so on. So there is no reason to conclude that the man has "always already" finished the countdown to zero.[18]

Again, Craig's response suggests that clearing up the confusion in the argument makes matters worse for the proponent of an infinite past:

> If we were to ask why the counter would not finish next year or in a hundred years, the objector would respond that prior to the present year an infinite number of years will have already elapsed, so that by

17. NMC, 157.
18. Wes Morriston, "Must the Past Have a Beginning?" *Philo* 2, no. 1 (1999): 5–19.

> the Principle of Correspondence, all the numbers should have been counted by now. But this reasoning backfires on the objector: for, as we have seen, on this account the counter should at any point in the past have already finished counting all the numbers, since a one-to-one correspondence exists between the years of the past and the negative numbers.[19]

Yet Craig makes a modal error here. Craig (and C&C in their article in NMC) asserts that the infinite counter necessarily must have finished counting "all" of the negative numbers prior to today. The fact is that a person counting for an infinite number of years could have reached zero yesterday, but it is not necessarily the case. Remember, the concept of "number" is equivocal when speaking of infinite numbers. The counter could also have reached −10 or −1,528 yesterday, or any other finite number. The infinite set of negative numbers consisting of {n… −13, −12, −11}, where there is no beginning term and ends in −11, or the set with no beginning term and ends in −1,528, or any other finite number for that matter, can also be put into a one-to-one correspondence with all past years because the number of terms in each of these sets is infinite. Thus, it is simply false that the counter must have finished counting all of the negative numbers prior to today. C&C confuse "all negative numbers" with "infinitely many negative numbers." The two sets are not necessarily the same. Yet Craig's argument works only if, necessarily, the counter must have counted all negative numbers by today. The argument is simply modally confused.

C&C are equally mistaken that an actual infinite past cannot be added to. Let's say that the counter finished counting yesterday with −11. That means that today he could count −10 and he will have added one term to the actual infinite. So an actual infinite can be added to. What about when he reaches 0—won't he have used all of the negative numbers? No matter, let him begin counting with 1 the next day, and 2 the day after, and so forth. Because all of the negative numbers can be put into a one-to-one correspondence with all negative and positive numbers together, continuing with positive numbers is continuing to count with the same order of infinity having cardinal number $\aleph_0$ and the order type ω_0.

19. William Lane Craig, "The Existence of God and the Beginning of the Universe," *Truth: A Journal of Modern Thought* 3 (1991).

But doesn't that leave some number in the infinite set unassigned? No. As Quentin Smith explains,

> To the objection that this [counting of negative numbers] leaves some previously past event without a negative number assigned to it there is the following response: Let us call the time before some instance of the above described reassignment t1, and the time of reassignment t2. At t2 there is a past event belonging to the collection of past events that had not belonged to this collection at t1. However, at t2 there is not a greater number of events belonging to this collection than at t1, for the addition of the one event at t2 to the infinite collection that had existed at t1 results in a collection the same number of members as the collection that existed at t1, this number being $\aleph_0$. This is true because $\aleph_0$ plus 1 equals $\aleph_0$. Consequently, since there are $\aleph_0$ past events at both times, and since there are $\aleph_0$ negative numbers, there is no past event at either time that is unmatched with a negative number.[20]

However, I want to point out a feature about counting that seems counterintuitive because our finite experience dictates that we must begin somewhere to begin counting. If an infinite or eternal counter has been counting without beginning, he need not start with any given finite number. When speaking of an infinite counter who counts without beginning, we define counting as merely a *synthetic act of marking new events by a count number*, and then it can make sense to think of each past event to correspond to an act of counting—and that is all that counting means when infinite series are involved. It seems that if I am counting that I must begin somewhere, and if I use "all" the negative numbers and reach 0 that somehow there are no more numbers to use. However, this argument confuses "all" when speaking of infinite, for there is no time at which there are not "more" numbers that can be assigned to mark any given event. (Remember, in this context, "more" means only "*can be placed in a one-to-one correspondence*".) An infinite counter can use all of the even and positive numbers in any order that he chooses to mark the one-to-one correspondence between past events and numbers. So long as no number is used twice, the correspondence of infinite sets holds. Thus, today I could use number 1,247,367,987,653 and tomorrow the number –3, and I can mark the days with any numbers that

20. Quentin Smith, "Infinity and the Past," in William Lane Craig and Quentin Smith, *Theism, Atheism, and Big Bang Cosmology* (Oxford, England: Clarendon Press, 1993), 85.

I wish to count as long as I don't use a number twice. The reason for this is that whichever finite number is used to add to an infinite is entirely arbitrary because they all have the same effect—a finite number added to an infinite number is always an infinite number.

This last point also answers the argument presented by J. P. Moreland and adopted by Francis Beckwith and Stephen Parrish, that the problem with having no beginning is that no one could even start counting:

> [A]ssume that someone had been counting toward zero from negative infinity from eternity past. If a person goes back in time from the present moment, he will *never* reach a point when he is finishing his count or engaging in the count itself. This is because at every point, he will have already had an infinity to conduct the count. As Zeno's paradox of the race course points out, the problem with such a situation is not merely that one cannot complete an infinite task; one cannot even start an infinite task from a beginningless situation.[21]

Now it is clear that the task of counting does not *begin* at some point if it has been taking place from all eternity. But that is no impediment for a person who lives in each moment of an eternal reality from marking by synthetic succession each new moment (assuming that moments are discrete). But surely the problem with a beginningless eternity cannot be the fact that it cannot have a beginning. Zeno's point from the race paradox is that Achilles cannot complete the journey because he cannot begin it. He cannot begin it because to do so he must complete one of the tasks that make up the journey; he must first complete another task and thus have already begun. But the claim that the past has no beginning cannot be refuted by arguing that a beginningless past could never begin—for that is just what the proponent of an infinite past claims.

4.0 A Beginningless Multiverse and Infinity

It is also important to see that the infinity arguments made by C&C do not apply to a beginningless universe of the type posited by the chaotic inflationary or quantum vacuum theories of cosmology. Let's take the chaotic inflationary theory first. According to this theory,

21. Quoted in Francis Beckwith and Stephen Parrish, *The Mormon Concept of God: A Philosophical Analysis* (Lewiston, N.Y.: The Edwin Mellen Press, 1991), 59.

our local space-time universe began a finite time ago. However, our local bubble universe is not all that there is. It is possible, and in fact predicted by the chaotic inflationary theory, that our universe arose from a prior universe that is not spatio-temporally continuous with our local universe.[22] This prior universe gave rise to our universe in the sense that it constitutes the conditions from which the singularity arises and from which our own local universe originated. Let's say that each bubble universe within the multiverse constitutes an epoch of a discrete space-time continuum. However, because a singularity or Big Bang event constitutes the beginning of any particular bubble universe, it follows that there is no continuous time metric between any two space-time epochs. This theory also predicts that the prior universe did not have the same initial constants as our local universe, and thus it is possible that there is no continuous time metric that is shared between the two epochs. In fact, there may be infinitely many separate bubble universes given chaotic inflationary theory.[23] However, each of these bubble universes has its own time metric that is shared by no other. Each has a beginning and possibly an end. Each is finite in the past. Nevertheless, the number of bubble universes in the multiverse may be infinite. However, because they are discontinuous spatio-temporal epochs, they do not constitute a *series* of any sort. There are no equal intervals of time that are shared between them. None of the infinity arguments presented by C&C (nor any that Craig has produced on his own) apply to an infinite number of discontinuous realities that do not form a series. All series of events in the chaotic inflationary theory are merely finite in duration. Thus, it is possible that the multiverse has always existed even though each of the bubble universes has only a finite past.[24] Moreover, it is possible that the multiverse has always existed even if, *arguendo*, any of the arguments made by C&C were sound.

Suppose that there had always existed a quantum vacuum of the type conceived in many current inflationary theories of cosmology. In

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22. Rem B. Edwards, "How Process Theology Can Affirm Creation *Ex Nihilo*," *Process Studies* 29, no. 1 (2000): 82.

23. See Andrei Linde, "The Self-Reproducing Inflationary Universe," *Scientific American* (Nov. 1994); "The Inflationary Universe," *Physics Today* 40 (1987): 61; and *Physics Review* 59 (1999); John D. Barrow, *Impossibility* (Oxford, England: Oxford University Press, 1998), 164–74; M.J. Rees, *Before the Beginning* (New York: Simon & Schuster, 1997).

24. John Barrow, *Impossibility*, 171–72.

this vacuum, there are innumerable events that occur within the limits of the Heisenberg Uncertainty Principle. None of these events are causally or temporally related to one another. They occur at random, and there is no time-metric to measure their proximity to one another.[25] Indeed, each of these events constitutes its own space-time universe in the sense that there is simply no causal or space-time continuum obtaining to place them in relation to one another. There is no beginning to this vacuum condition—it is simply the lowest energy state possible in the physical world. Such a reality must be regarded as quiescent in the sense that no events give rise to a series of events until the vacuum decays into a false vacuum creating the energy from which the Big Bang derived. Craig has admitted, correctly in my view, that the infinity arguments do not apply to a quiescent universe (i.e., a physical reality having no events).[26]

What Craig fails to address is how any of the arguments he (or he and Copan) presents could apply to the quantum vacuum. All of his arguments assume the existence of a well-formed infinite series. That is, a series that has *consecutive terms*, or has a *next term* after any selection of terms, like {1/2, 1/4, 1/8, 1/16, 1/32 . . .}. Because the quantum vacuum does not have a next term after any selection of terms, it is not a well-formed series. Indeed, it is simply no series at all but only unconnected random events—the ultimate description of chaos. Nevertheless, it is an eternal reality that has no beginning. Craig admits that the infinity arguments cannot demonstrate the existence of the universe out of nothing given the possibility of a quiescent universe: "*Creatio ex nihilo* would not then be proved, but as I employ it the *kalam* cosmological argument's primary aim is to support theism, not *creatio ex nihilo*."[27] However, it is not just a quiescent universe that escapes the arguments, but also any reality that is not continuous. Therefore, the infinity arguments also do not apply to a multiverse that has no beginning and has always existed as a quantum vacuum.

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25. Alan H. Guth, *The Inflationary Universe* (New York: Addison Wesley, 1998), 167–87, 245–52; Timothy Ferris, *Coming of Age in the Milky Way* (New York: William Morrow, 1988), 349–66.

26. Craig, "The 'Kalam' Cosmological Argument and the Hypothesis of a Quiescent Universe," 104–8.

27. Ibid., 106–7.

Perhaps it could be argued that even if the events in the quantum vacuum do not constitute a well-formed series, they nevertheless still form an infinite *collection*. Or, even if the bubble universes does not constitute a spatio-temporally continuous reality, they still create a collection related in some sense to the cause-effect relation—in the sense that a new universe can be explained by conditions in the prior universe. Nevertheless, our concept of "cause-and-effect" derives from our experience in this spatio-temporal epoch. Look at the arguments presented by C&C. The second argument is based on a *series* that is formed by successive addition. A mere collection of random events that are not additive to one another won't work. Certainly the events of the quantum vacuum do not form anything by successive addition. Further, since the bubble universes posited in the chaotic inflationary theory do not constitute spatio-temporally related realities, it is difficult to see how the notion of addition as a series can be applied. It is also difficult to see how premise 1.1 of the first argument could be supported, because none of the supposed absurd stories apply to discontinuous realities. None of the thought experiments like Hilbert's Hotel could apply to either the vacuum or the bubble universes because they cannot be, as unrelated events, manipulated, reversed, halved or emptied the way the rooms in Hilbert's Hotel are. Thus, even if the arguments given to show that an actually infinite series were somehow sound, they don't apply to the multiverse envisioned in the chaotic inflationary theory.

5.0 Logical Possibility and the Uncreated Universe

C&C don't claim that a contradiction in first order logic can be derived from the proposition that the universe is not created and thus without a beginning. What they claim is that the idea is absurd. They thus claim that the notion of an infinitely old universe is metaphysically impossible—that is, there is no possible world in which such a universe can exist. Yet all C&C really mean by "logically impossible" is that they think the notion of an actual infinity is absurd even though they cannot show an outright contradiction in the notion. I find nothing absurd at all about the notion of an actually infinite past, though the results of transfinite mathematics are strange, given my expectations based upon experiences with finite realities. However, while the results of transfinite numbers and the concept of an infinite universe may be strange, they are not any stranger than the notions of quantum mechanics or the theory of relativity. We have learned from scientific breakthroughs that the universe is a strange place that often conflicts with our expectations and experience. Indeed, we encounter realities that are so strange that we can't even accurately picture them. A universe where an event is simultaneously a particle and a wave, or a reality that literally does not have both position and momentum at once, or space that bends and curves, or clocks that run faster and slower depending on the inertial frame of reference of the observer are at least as strange as anything we encounter dealing with infinities. C&C seek to exploit this strangeness to convince us that a universe that is eternal is simply absurd, but once the behavior of transfinite numbers and infinite realities is grasped, they are not strange but exciting and mind-expanding.

If the eternal universe is really impossible, let me ask just how old it is *logically possible* for the universe actually to be? Is there some largest number than which C&C would claim the actual universe could not be older? Of course not. The reason why C&C will not give us a largest number for the possible age of the universe is obvious—there is no such largest number at which the universe could not be older.

This recognition is no trivial matter, for it follows from the fact that there is no largest number that the set constituting the number of times at which it is logically possible that the universe actually existed has the same properties as the set of all negative numbers. Consider a thought experiment. Let's say that I have a time machine that will let me visit any time in the past at which it is logically possible that the universe *actually* existed. There may be physical barriers to the number of times that I can visit—for example, I may not be able to traverse a Big Bang event. If I cannot travel back in time past the circumstances obtaining in the early local universe, then the number in the set of past moments that I can visit is *physically* limited. However, since I believe that time travel is physically and nomologically impossible anyway, I am not speaking of natural or physical possibility. Because I want to talk about *logical possibility* and not merely what is permitted by natural laws, let me stipulate that my time machine can survive any Big Bang event. Moreover, I am not asking about whether visiting a moment is merely logically possible, but in visiting any past moment at which it is logically possible that the universe actually existed. Is there some limit to the number of times that I could visit in which it is logically possible that the physical universe *actually existed*?

Let's say that I set the clock in the time machine so that every 30 seconds I visit a past year in geometric progression to the geometric power beginning with a year two years ago. So I first visit 2 years ago in the first 30 seconds, and then I visit 4 to the fourth power (4^4) years ago after 60 seconds, and then I visit 16 to the sixteenth power years ago after 90 seconds, and so forth. After a mere ten minutes, I have visited times well older than the time of the Big Bang—about 6.6 billion years ago. But it is logically possible that the universe is older than that. After a mere hour, I have gone so far into the past that if all the 0s (just the size of the zeros on this page) needed to write the number were written on a normal piece of paper, they would fill more than the volume of the entire known universe! And yet I can still travel back in time, because for every time I pick, it is logically possible that the universe actually existed at that time. Imagine how far back I have gone after just one day. The point of this thought experiment is that there is no largest number, and so no limit to the times I could visit. No matter how far back in time I go, it is always logically possible that the world actually existed at that time.

Now let's modify the thought experiment just a bit. Let's say that instead of choosing a time further and further into the past, all I do is randomly choose various moments in the past to visit. When deciding a past time to visit, I am presented an array of possibilities from which to choose. I want to limit my choices to those times or moments in the past when it is logically possible that the world actually existed. Now let's ask the crucial question: Just how large is *the set of past times* from which I can choose to visit at which it is logically possible that the universe actually existed? A moment's reflection will show that this set is unlimited and in fact has the same properties as the complete set of real numbers—the members of this set can therefore be put into a one-to-one correspondence with the completed set of real numbers, which is infinite. The set of real numbers has $\aleph_0$ members. It follows that the number of past times at which it is possible that the universe actually existed has $\aleph_0$ members. Thus, the set of past times that are possible for me to visit and at which it is logically possible that the universe actually existed is also infinite.

Let's call *the set of past times at which it is logically possible that the universe actually existed* set *Sp*. The argument is as follows:

3.1 The members of the set *Sp* can be placed into a one-to-one correspondence with the members of the set of real numbers.
3.2 Sets whose members can be placed into a one-to-one correspondence with one another have the same number of members.
3.3 The set of real numbers has $\aleph_0$ members.
3.4 Therefore, set *Sp* has $\aleph_0$ members.

This argument is valid. Moreover, the premises seem unassailable. The only real possible question is whether the members of set *Sp* can be placed into a one-to-one correspondence with the set of real numbers as asserted by 3.1. Now it is clear that if I *begin* to visit past times, the set of times that I will have actually visited will always be finite. However, I am not inquiring about the set of times I can actually visit *by beginning to visit past times* but how large is the set of past times from which I can choose to visit? It is this set of past times *Sp* that I *could choose to* visit that has $\aleph_0$ members. Because the set is unlimited, it is logically possible that the world has always actually existed.

Moreover, there is in fact a way that I could visit an infinite number of times. If I had always been visiting times as they occurred without a beginning, then I will have visited an infinite number of times. Thus, it is not the time machine that allows me to visit the infinite number of times from which I could choose; rather, only by visiting each time as it actually occurs could I ever visit an infinite number of past times. Only one type of being can visit all of the times in an infinite past—an eternal being that actually existed in each of those times.

C&C reject this argument. They state,

> Ostler thinks that because the number of possible worlds with longer and longer finite pasts is unlimited, therefore there is a possible world having an infinite past. This is logically fallacious as reasoning that because one can count higher and higher finite numbers without limit, therefore there must be an infinitieth number.[28]

Yet C&C have misstated and misconstrued the argument. First, nothing in my argument suggests that *the set of past times* at which it is

28. NMC, 159. I provided a preliminary draft to Craig that did not contain the second step of the argument, and this failure on my part may have led to confusion about what my argument actually was.

logically possible that the universe actually existed contains a *particular member* that occurred an infinite number of moments ago. I am not arguing for a set of possible worlds that has longer and longer finite pasts as C&C claim. Indeed, my argument no more suggests an infinitieth number than the set of real numbers being infinite implies that there is an infinitieth real number. Any particular past time is, in fact, a finite number of intervals away from the present. But C&C themselves commit the fallacy of composition by suggesting that if the individual members of the set of past times are all finite, then *the set* of all past times *Sp* must also be finite! Because I argue that the *set of past times* has the property of infinity rather than any of its individual members, C&C simply manufacture a fallacy where there is none, and by so doing they commit the very fallacy of composition that they attribute to my argument.

Moreover, note that I say nothing about possible worlds semantics in my argument. Rather, what I argue is that the set of past times at which it is logically possible that the universe actually existed has the same properties as the complete set of negative numbers. The negative numbers can be placed in a one-to-one correspondence with the times it is possible that the universe actually existed. It follows that the *set* of past times at which the world actually exists also has an infinite number of members. I am not arguing that there is a possible world that has the property of having always existed. Instead, I am making a modal claim that is true in *all* possible worlds. In every possible world, the set of past times at which it is possible that the world actually existed is infinite. My argument shows that it can be demonstrated that it is logically possible that the world has no beginning because the set of past times at which it is logically possible that the world actually existed is infinite.

Finally, it should be noted that because this arguments deals with the times at which it is logically possible that the world actually existed, it bypasses concerns about whether the infinity arguments must be *a posteriori* (empirical) or can be merely *a priori*, that is, whether a reality exists in the actual world as an empirical or *a posteriori* question that is decided by experience, and not merely by whether we have a concept of it. C&C want to discuss the ontological status of infinities in the real world and not merely their conceptual status. Indeed, they reject the Platonic view that numbers and mathematical entities are real. If they admitted the Platonic view, then they would have to admit that not merely are infinities logically consistent, but also that infinities also *actually occur*. Indeed, it seems that there is in fact evidence that infinities actually occur in the real world because infinities turn up in standard quantum mechanical equations that give accurate predictions of quantum effects in the real world. Yet because C&C deal with actual infinities rather than with conceptual infinities, it is confusing to see how they can reach any conclusions based on a discussion of concepts and thought-experiments rather than empirical data.[29] My argument does not attempt to establish that infinities actually do or do not occur in the real world but instead demonstrates that it is logically possible that the world has always existed.

6.0 Conclusion

Copan and Craig have not given us any reason to believe that an eternal reality is either physically or logically impossible. The first argument commits the fallacy of equivocation. None of the supposedly absurd stories applies to the eternal universe. The second argument has two false premises. Those who believe in an infinite past do not claim that it can be formed by successive addition; in fact, they claim that it is in the nature of such realities that the concept of formation by successive addition doesn't apply. Thus, premise 2.1 is false. Moreover, the notion that it is impossible to add to an actual infinite is simply in error. Thus, premise 2.2 is also false.

Even if the arguments were sound, *per impossibile*, they would not apply to discontinuous spatio-temporal epochs such as those posited by the chaotic inflationary and quantum vacuum theories of cosmology. However, these theories must be considered to be speculative metaphysics rather than empirical science. I am speaking of possibilities, opening new horizons for consideration rather than dogmatically asserting that reality is actually structured as these theories predict. However, the recognition that it is logically possible that the world has al-

29. See the discussion in Graham Oppy, "Reply to Professor Craig," Sophia 34, no. 2 (December 1995), 15–29; "Craig, Mackie, and the Kalam Cosmological Argument," Religious Studies 27 (June 1991): 189–97; John Taylor, "Kalam: A Swift Argument from Origins to First Cause?" Religious Studies 33, no.2 (167–79); John L. Mackie, "The Miracle of Theism" (Oxford, England: Oxford University Press, 1982).

ways existed is not insignificant. It shows that it is not possible for the arguments suggesting otherwise to be constructed successfully.